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Statistical Thinking in Sports

CRC PRESS Boca Raton Ann Arbor London Tokyo

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## 13

## Myths in Tennis

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#### Abstract

Many people have ideas about tennis. In particular, most commentators hold strong ideas about, for example, the advantage of serving first in a set, the advantage of serving with new balls, and the special ability of top players to perform well at the "big" points. In this chapter we shall investigate the truth (more often the falsity) of a number of such hypotheses, based on Wimbledon singles' data over a period of four years, 1992–1995.

#### 13.1 INTRODUCTION

The All England Croquet Club was founded in the Summer of 1868. Lawn tennis was first played at The Club in 1875, when one lawn was set aside for this purpose. In 1877 The Club was retitled The All England Croquet and Lawn Tennis Club and the first tennis championship was held in July of that year. Twenty-two players entered the event which consisted of men's singles only. Spencer Gore became the first champion and won the Silver Challenge Cup and twelve guineas, no small sum (about £ 800 in today's value), but rather less than the £ 655000 that the 2006 champion Roger Federer received. In 1884 the women's singles event was held for the first time. Thirteen players entered this competition and Maud Watson became the first women's champion receiving twenty guineas and a silver flower basket. (William Renshaw, the 1884 men's singles champion received thirty guineas.) In 1922 The Championships moved from Worple Road to its current location at Church Road; see Riddle (1988) and Little (1995) for some historical details. For more than a century The Championships at Wimbledon have been the most important event on the tennis calendar. Currently both the men's singles and the women's singles event are restricted to 128 players.

Because of television broadcasts, tennis has become a sport which is viewed by millions all over the world. Many have ideas about tennis. In particular, most commentators hold strong ideas about, for example, the advantage of serving first in a set, the advantage of serving with new balls, and the special ability of top players to

perform well at the "big" points. In this chapter we shall investigate the truth (more often the falsity) of a number of such hypotheses.

In Section 13.2 we discuss the data and some selection issues. Our data are obtained from Wimbledon singles' matches over a period of four years, 1992–1995. Further data at this level of detail were not available to us. In Section 13.3 we discuss three popular myths concerning the service and show that all three are false. The first two myths were earlier discussed in Magnus and Klaassen (1999c), and the third in Magnus and Klaassen (1999a). Section 13.4 discusses the notion of "winning mood" (dependence, in statistical parlance) by considering the final set and breaks. The two myths about the final set were analyzed in Magnus and Klaassen (1999b), the two myths about breaks are new results. Section 13.5 concerns "big points" (that is, identical distribution for the statistician); these results are also new.

We shall see that almost all hypotheses are rejected, but not all. It is not true that serving with new balls or starting to serve in a new set are advantages. Also, the seventh game is not especially important. But, it is true that big points exist and that real champions perform their best at such points.

#### 13.2 THE DATA AND TWO SELECTION PROBLEMS

We have data on 481 matches played in the men's singles and women's singles championships at Wimbledon from 1992 to 1995. This accounts for almost half of all singles matches played during these four years. For each of these matches we know the exact sequence of points. We also know at each point whether or not a second service was played and whether the point was decided through an ace or a double fault. Table 13.1 provides a summary of the data. We have slightly more matches for men than for women, but of course many more sets, games, and points for the men's singles than for the women's singles, because the men play for three sets won and the women for two. The men play fewer points per game than the women, because the dominance of their service is greater. But the women play fewer games per set on average—scores like 6–0 and 6–1 are more common in the women's singles than in the men's singles-because the difference between the 16 seeded and the 112 nonseeded players is much greater. (Until 2000, 16 players out of 128 were seeded at Wimbledon, thereafter 32. Seeded players receive a protected placing on the schedule, so that they cannot meet in the early stages of the tournament. Typically, but not necessarily, the top players in the world rankings are seeded.) This also leads to fewer tie-breaks in non-final sets for women. (By "final set" we mean the fifth set in the men's singles and the third set in the women's singles. At Wimbledon there is no tie-break in the final set.) Both men and women play about 60 points per set. The men play on average 230 points per match, the women 132, and hence a match in the men's singles takes on average 1.75 times as long as a match in the women's singles.

All matches in our data set are played on one of the five "show courts': Centre Court and courts 1, 2, 13, and 14. Usually matches involving top players are scheduled on these courts. This causes an under-representation in the data set of matches involving non-seeded players. This is, however, not the only selection problem. If two non-seeded players play against each other in the quarter-final, this match is

#### data set. Women's singles Men's singles Matches 258 223 Non-final sets 899 446 Final sets 51 57 9367 Games 4486 Tie-breaks 177 37 Points 59466 29417 Averages 3.7 2.3 Sets in match Games in non-final set 9.8 8.9 Games in final set 9.2 11.1Tie-breaks in non-final set 0.2 0.1 131.9 Points in match 230.5 Points in game 6.5 6.1 Points in tie-break 12.111.8

**TABLE 13.1**Number of matches, sets, games, tie-breaks, and points in the<br/>data set.

likely to be scheduled on a show court. But, if they play in the first round, their match is considered to be of less importance and is likely to be played on another court. After all, there are 16 first-round matches involving a seeded player and such matches usually take precedence. Therefore, the under-representation of matches between two non-seeded players is most serious in early rounds. This dependence on round in the selection of matches is also present in other types of matches, although it is less serious, as Table 13.2 shows.

We distinguish between round (1, first round; 7, final) and type of match (Sd-Sd for two seeded players, Sd-NSd for a seeded against a non-seeded player and NSd-NSd for two non-seeded players). The column labeled "Sam" in each part contains the number of matches in our sample, and the column labeled "Pop" the number of matches actually played. (Note that in the first round of the women's singles there are 63 rather than 64 matches between a seeded and a non-seeded player. The reason is that Mary Pierce, seeded 13, withdrew in 1993 at the last moment. She was replaced by Louise Field, an unseeded player.)

We see that the percentage of matches of non-seeded against non-seeded (NSd-NSd) players in our data set is 24.9 (74/297) for the men and 14.8 (42/283) for the women. Both are lower than the percentages for Sd-NSd matches, which are themselves lower than those for Sd-Sd matches. This illustrates the first selection problem, namely the under-representation of matches involving non-seeded players.

The second selection problem, caused by the round dependence, results from the increasing pattern in the sampling percentages over the rounds. For example, only 32.0% (82/256) of all first-round matches in the men's singles and 26.2% (67/256) in the women's singles are in the data set, whereas all finals have been sampled.

Round	Sd-	Sd	Sd-1	NSd	NSd-	NSd	То	tal
	Sam	Pop	Sam	Pop	Sam	Рор	Sam	Рор
(a) Men	's singl	les						
1	—	—	48	64	34	192	82	256
2		—	46	54	16	74	62	128
3	_	—	39	41	16	23	55	64
4	8	9	15	15	8	8	31	32
5	7	7	9	9	0	0	16	16
6	7	7	1	1	0	0	8	8
7	4	4	0	0	0	0	4	4
Total	26	27	158	184	74	297	258	508
(b) Won	ıen's si	ngles						
1	—	—	43	63	24	193	67	256
2	—	—	43	58	3	70	46	128
3	—	—	42	48	12	16	54	64
4	8	8	20	21	2	3	30	32
5	11	12	3	3	1	1	15	16
6	6	6	1	2	0	0	7	8
7	4	4	0	0	0	0	4	4
Total	29	30	152	195	42	283	223	508

**TABLE 13.2**Number of matches in the sample (Sam) and in the population(Pop).

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Since we wish to make statements about Wimbledon (and not just about the matches in our sample), we account for both selection problems by weighting the matches when computing the statistics in this chapter. The weights are calculated as the ratios Pop/Sam in Table 13.2. This procedure involves an assumption, namely that within each cell the decision by Wimbledon's organizers whether a match is on a show court or not is random, so that the matches on the show courts (which are the matches that we observe) are representative. One could argue that, if the sample is very small compared with the population, this method would make the few observed matches too important. Most notably, in the womens singles we observe only three of the 70 matches played between two non-seeded players in the second round. If these three matches were selected by the organizers to include, for example, players just outside the top 16, then our method would be seriously biased for this cell. As it happens, the three matches concern players with Women's Tennis Association rankings 27-41, 131-143 and 22-113 and hence there is no reason to believe that these matches are not representative.

#### **13.3** SERVICE MYTHS

The service is one of the most important aspects of tennis, particularly on fast surfaces such as the grass courts at Wimbledon. In Table 13.3 we provide some of its characteristics. As before, Sd-NSd indicates a match of a seeded against a non-seeded player, where the first player (Sd) is serving in the current game and the second player (NSd) receiving. Sd-Sd, NSd-Sd, and NSd-NSd are similarly defined. Standard errors are given in parentheses. To obtain the standard errors, we have treated all points as independent. This is not quite true (see Klaassen and Magnus, 2001): a good point (for example, an ace) may bring about another good point, and a bad point (missed smash) may bring about another bad point; but it is sufficient as a first-order approximation for our purpose.

We see that, at Wimbledon, the men serve almost three times as many aces as the women but serve the same number of double faults. (The percentage of aces is defined as the ratio of the number of aces (first or second service) to the number of points served, rather than to the number of services.) In understanding the other service characteristics in Table 13.3, the distinction between "points won if 1st (2nd) service in" and "points won on 1st (2nd) service" is important. In the men's singles, when two seeded players play against each other, the first service is in 58.7% of the time. If the first service is in, the probability of winning the point is 77.7%. Therefore, in the men's singles the probability of winning the point on the first service is 58.7%  $\times$  77.7% = 45.6%; see the second column of Table 13.3, part (a). Hence,

% points won on 1st service

= (% points won if 1st service in)  $\times$  (% 1st services in), (13.1)

and, of course, the same for the second service. Combining the data for the first and second services, we can derive the percentage of points won on service. A player can win a point on service in two ways: on the first or on the second service. However,

Service characteristics.							
Characteristic	% of the characteristics for:						
	Sd-Sd	Sd-NSd	NSd-Sd	NSd-NSd	Total		
(a) Men's singles							
Aces	11.7	11.0	7.7	7.0	8.2		
11005	(0.4)	(0.2)	(0.2)	(0.2)	(0.1)		
Double faults	5.1	5.1	5.8	5.6	5.5		
	(0.3)	(0.2)	(0.2)	(0.2)	(0.1)		
Points won on service	67.0	<u>69.3</u>	61.1	63.7	64.4		
	(0.6)	(0.4)	(0.4)	(0.4)	(0.2)		
1st services in	58.7	59.6	59.4	59.5	59.4		
	(0.6)	(0.4)	(0.4)	(0.4)	(0.2)		
2nd services in	87.8	87.3	85.6	86.2	86.4		
	(0.6)	(0.4)	(0.4)	(0.4)	(0.2)		
Points won if 1st service in	77.7	78.1	70.2	72.4	73.3		
	(0.7)	(0.4)	(0.4)	(0.5)	(0.2)		
Points won if 2nd service in	59.0	64.5	55.6	59.2	59.4		
	(1.0)	(0.6)	(0.6)	(0.6)	(0.3)		
Points won on 1st service	45.6	46.5	41.7	43.1	43.6		
	(0.6)	(0.4)	(0.4)	(0.4)	(0.2)		
Points won on 2nd service	51.8	56.3	47.6	51.0	51.4		
	(0.9)	(0.6)	(0.6)	(0.6)	(0.3)		
Games won on service	86.0	88.9	74.1	79.7	80.8		
	(1.1)	(0.6)	(0.8)	(0.8)	(0.4)		
(b) Women's singles							
Aces	3.3	4.2	2.5	2.9	3.1		
	(0.3)	(0.2)	(0.2)	(0.2)	(0.1)		
Double faults	3.9	4.2	5.8	6.0	5.5		
	(0.3)	(0.2)	(0.2)	(0.3)	(0.1)		
Points won on service	56.9	62.9	50.1	55.8	56.1		
	(0.8)	(0.5)	(0.5)	(0.7)	(0.3)		
1st services in	65.5	61.5	60.5	60.2	60.8		
	(0.7)	(0.5)	(0.5)	(0.7)	(0.3)		
2nd services in	88.8	89.1	85.2	85.1	86.0		
	(0.8)	(0.5)	(0.6)	(0.8)	(0.3)		
Points won if 1st service in	62.5	69.6	56.4	61.7	62.2		
	(0.9)	(0.6)	(0.6)	(0.8)	(0.4)		
Points won if 2nd service in	51.8	58.6	47.4	55.2	54.1		
	(1.4)	(0.9)	(0.9)	(1.1)	(0.5)		
Points won on 1st service	41.0	42.8	34.1	37.1	37.8		
	(0.8)	(0.5)	(0.5)	(0.6)	(0.3)		
Points won on 2nd service	46.0	52.2	40.4	4/.0	46.6		
Comes was an emile	(1.3)	(0.8)	(0.8)	(1.1)	(0.5)		
Games won on service	00.3	//.ð	49.4	02.8	03.4		
	(1.9)	(1.1)	(1.5)	(1.6)	(0.7)		

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TABLE 13.3

the second possibility only becomes relevant when the first serve is a fault. Thus,

% points won on service = % points won on 1st service

+ (% 1st service not in)  $\times$  (% points won on 2nd service). (13.2)

For example, from the second column of Table 13.3, part (a),

$$67.0\% = 45.6\% + (100 - 58.7)\% \times 51.8\%.$$

The literature on the service in tennis concentrates on the first/second service strategy rather than on myths relating to the service. In Gillman (1985) it is suggested that "missing more serves may win more points;" see also Gale (1980), George (1973), Hannan (1976), and Norman (1985). Borghans (1995) analyzed the 1995 Wimbledon final between Sampras and Becker, and showed that Becker could have performed much better if he had put more power in his second service. In a recent paper Klaassen and Magnus (2006) we ask whether the service strategy is efficient and measure its inefficiency.

#### 13.3.1 A PLAYER IS AS GOOD AS HIS OR HER SECOND SERVICE

Many commentators say this, but is it true? Let us first ask how to measure the quality of the second service. This is not the percentage of second services in nor the percentage of points won if the second service is in, but it is the combination of the two, namely the percentage of points won on the second service. In the men's singles, let us compare the matches Sd-Sd and NSd-NSd; see the second and fifth columns in Table 13.3, part (a). With some simplification, the players in these matches can be considered to have the same strength: they are either both good (NSd-NSd) or both very good (Sd-Sd). We see that the seeded players win significantly more points on their first service than do the non-seeded players (45.6% > 43.1%), but that the estimated probabilities of winning points on the second service are not significantly different. (Throughout this chapter we use a 5% level of significance.)

Hence a seeded player distinguishes himself from a non-seeded player by having a better first service, not by having a better second service. Therefore, the idea that "a player is as good as his or her second service" is not supported by the Wimbledon data; The same holds in the women's singles as can be verified from part (b) of Table 13.3.

There is, however, one important difference between the men's singles and the women's singles and this relates to the quality of the first service. Referring to Equation (13.1), the quality of the first service is made up of two components: the percentage of points won if the first service is in and the percentage of first services in. In the men's singles the difference in the quality of the first service between seeded and non-seeded players is determined primarily by the percentage of points won if the first services in is not significant. In the women's singles the difference is not significant. In the women's singles the difference is determined primarily by the percentage of first services in (65.6% is significantly larger than 60.2%), whereas the difference in the percentage of points won if first service is in is insignificant.

We conclude that it is not true that "a player is as good as his or her second service." It would be more realistic to say that "a player is as good as his or her *first* service." The first service is more important than the second, but the aspect of the first service which matters differs between men and women.

#### 13.3.2 SERVING FIRST

Most players, when winning the toss, elect to serve. Is this a wise strategy? This depends on whether or not you believe that there exists a psychological advantage to serve first in a set. This idea is based, presumably, on the fact that the player who receives in the first game is usually one game behind and that this would create extra stress. The advantage, if is exists, can only be psychological, because there is no theoretical advantage (Kingston, 1976; Anderson, 1977). Let us investigate whether there is any truth in this *idée reçue*.

Our first calculations seem to indicate that the idea must be wrong. Overall only 48.2% of the sets played in the men's singles are won by the player who begins to serve in the set. In the women's singles the percentage is 50.1%. The standard errors of the two estimates are 1.6% and 2.2%, respectively. Therefore, neither of the two percentages is significantly larger than 50%. If we look at the sets separately, then we see that this finding (starting to serve provides no advantage) seems to be true in every set, except perhaps the first. In the men's singles the estimated probability of winning a set when starting to serve is 55.4% (3.1%) in the first set and 44.3% (3.1%), 43.5% (3.1%), 51.0% (4.5%) and 48.8% (7.0%) in the second to fifth sets respectively. In the second and third sets serving first may even be a disadvantage.

Exactly the same occurs in the women's singles. There the probability that the player who starts to serve also wins the set is estimated to be 56.6% (3.3%) in the first set, 44.0% (3.3%) in the second set and 47.8% (6.6%) in the third set. Apparently starting to serve in a set is a *disadvantage* rather than an advantage, except perhaps in the first set.

This is a little puzzling. We can accept perhaps that there is no advantage in serving first, by why should there be a disadvantage and why should this disadvantage only exist in second and following sets, but not in the first? Let us take a closer look.

Consider a match between a stronger player and a weaker player, and consider the beginning of the second set. It is likely that the stronger of the two players has won the first set; after all he/she is the stronger player. Also, it is likely that the last game of the previous set has been won by the server in that game. As a result, the stronger player is likely to have served out the set, and hence the weaker player will (usually) start serving in the second set. The probability that the player who starts to serve in the second set will more often lose than win the set may therefore be due to the fact that it is typically the weaker player who starts to serve in the second set. The same holds for all sets, except the first. This explains that in all sets except the first the percentages are less than 50%, not because there is a disadvantage for the player who serves first in a set but because the server in the first game is usually the weaker player. A proper analysis should take this into account. and this calls for a conditional rather than an unconditional analysis.

**TABLE 13.4** Estimated probabilities of winning a set after winning the previous set (*S*: starts serving; *R*: starts receiving).

Set	Sd-	Sd	Sd-NSd		NSd-Sd		NSd-NSd		Total	
•	S	R	S	R	S	R	S	R	S	R
(a) N	Aen's sing	gles								
1	46.8	53.2	80.6	78.9	21.1	19.4	56.9	43.1	55.4	44.6
	(9.8)	(9.8)	(4.2)	(4.9)	(4.9)	(4.2)	(5.8)	(5.8)	(3.1)	(3.1)
2	61.0	52.6	82.1	78.8	19.3	30.8	72.1	70.0	72.5	68.0
	(21.8)	(10.9)	(6.0)	(4.4)	(16.1)	(9.1)	(9.0)	(6.6)	(5.1)	(3.5)
3	79.5	75.2	73.5	76.7	64.2	40.5	75.0	73.6	73.9	72.1
	(12.8)	(10.8)	(6.7)	(4.7)	(13.8)	(10.5)	(9.2)	(6.1)	(4.7)	(3.4)
4	19.5	50.0	74.4	69.8	26.2	34.3	75.9	68.8	62.9	60.2
	(17.7)	(14.4)	(11.3)	(10.3)	(10.1)	(10.4)	(10.4)	(11.6)	(6.5)	(5.9)
5	0.0	11.9	70.5	86.1	24.8	37.1	52.8	60.6	48.3	51.0
	(0.0)	(11.5)	(26.3)	(13.1)	(21.6)	(13.4)	(17.7)	(18.5)	(12.5)	(8.5)
(b) V	Vomen's s	ingles								
1	62.4	37.6	72.4	84.0	16.0	27.6	64.3	35.7	56.6	43.4
	(9.0)	(9.0)	(5.1)	(4.3)	(4.3)	(5.1)	(7.4)	(7.4)	(3.3)	(3.3)
2	64.3	72.6	89.5	90.3	27.2	31.9	69.5	74.3	72.0	75.2
	(14.5)	(10.5)	(4.3)	(3.6)	(12.3)	(10.2)	(10.3)	(9.3)	(4.6)	(3.8)
3	40.4	25.0	67.5	85.9	0.0	15.0	92.9	60.8	63.5	60.1
	(21.9)	(21.7)	(14.8)	(9.7)	(0.0)	(13.5)	(12.9)	(17.3)	(9.6)	(8.9)

In part (a) of Table 13.4 we consider a player in the men's singles *who has won the previous set* and compare the estimated probability that he wins the current set when starting to serve with the estimated probability that he wins the current set when starting to receive. For example, if a seeded (Sd) player has won the first set against a non-seeded (NSd) player, then his probability of winning the second set is estimated as 82.1% when he (the seeded player) begins to serve and as 78.8% when his opponent begins to serve. Of course, there is no set before the first and hence the probabilities in the first row are simply the (unconditional) probabilities of winning the first set. The same probabilities, estimated for the women's singles, are provided in part (b) of Table 13.4.

Let us consider the first three sets in the men's singles and the first two sets in the women's singles, because the other sets have relatively few observations and hence large standard errors. There is some indication that in the men's singles there is an advantage in serving first: the overall probability of winning a set after winning the previous set is higher for the player who begins to serve than for the player who begins to receive. The difference is 10.8 percentage points (6.2%) in the first set, 4.5 percentage points (6.2%) in the second set and 1.8 percentage points (5.8%) in the third set. However, these results are not significant and the differences are not positive for all four subcategories (Sd-Sd, Sd-NSd, etcetera). We conclude that the support for our hypothesis is insufficient, except perhaps in the first set.

In the women's singles the first set indeed appears to be special. The probability of winning the first set is significantly higher for the player who begins to serve than for the player who begins to receive (the difference is 13.2 percentage points on average with a standard error of 6.6%). The probability of winning the second set

after winning the first is lower, although not significantly, for the player who begins to serve than for the player who begins to receive (the difference is 3.2 percentage points with a standard error of 6.0%). Hence in the women's singles our hypothesis is only true for the first set. This implies that electing to serve after winning the toss is generally better than electing to receive.

There is only one result in this analysis that is reasonably robust and that is that in the first set of a match there is an advantage in serving first. But why should the first set be different from other sets? Maybe this is because fewer breaks occur in the first few games of a match? Yes, but we can be more precise. The reason why the probability in the first set is higher is entirely due to the effect of the first game in the match. The probability that the server wins this first game is 87.7% (2.0%) rather than 80.8% in the men's singles and 74.3% (2.9%) rather than 63.4% in the women's singles; see Table 13.3. It is only the very first game that is special. In the second game the percentages of winning a service game are not significantly different from those concerning the first set excluding the first two games. This holds for both men and women.

Hence, what to do when you win the toss? Is it wise—as sometimes argued—to elect to *receive* when you win the toss, because it is easier to break your opponent in the first game of the match than in later games. In the first game, the argument goes, the server is not yet playing his or her best tennis. If this is true, however, then it must be that the receiver is performing even worse in the first game, as we find that breaks in the first game occur less often. Therefore, it is wise to elect to *serve* when you win the toss.

#### 13.3.3 NEW BALLS

Tennis as a game has a long history which goes back to the Greeks and Romans. But it was not until 1870 that it became technically possible to produce rubber balls which bounce well on grass. When the All England Lawn Tennis Club decided to hold their first championships in 1877, a three-man subcommittee drew up a set of laws. Rule II stated that

"the balls shall be hollow, made of India-rubber, and covered with white cloth. They shall not be less than 2 1/4 inches, nor more than 2 5/8 inches in diameter; and not less than 1 1/4 ounces, nor more than 1 1/2 ounces in weight." (Little, 1995, p. 284)

The quality of the tennis balls has gradually improved. From 1881 to 1901 the balls were supplied by Ayres; thereafter by Slazinger and Sons. Yellow balls were introduced at the 100th championships meeting in 1986. During the 1877 championships 180 balls were used; now more than 30000 are used in one year.

During a tennis match new balls are provided after the first seven games (to allow for the preliminary warm-up) and then after each subsequent nine games. Most commentators and many spectators believe that new balls are an advantage to the server. But is this true?

To determine whether serving with new balls is an advantage, let us consider Table 13.5. The age of the balls in games is indicated from 1 (new balls) to 9 (old balls).

TABLE 13.5										
Service characteris	stics de	ependi	ng on t	he age	of the	balls.				
Characteristic	Pe	Percentage of the characteristics for the following ages of balls:								
	1	2	3	4	5	6	7	8	9	
(a) Men's singles										
Aces	8.7	7.9	8.6	7.7	8.2	8.3	8.5	8.4	7.2	8.2
	(0.4)	(0.4)	(0.3)	(0.3)	(0.3)	(0.3)	(0.3)	(0.4)	(0.3)	(0.1)
Double faults	5.8	5.3	5.8	6.6	5.4	5.6	4.9	5.1	5.1	5.5
	(0.3)	(0.3)	(0.3)	(0.3)	(0.3)	(0.3)	(0.3)	(0.3)	(0.3)	(0.1)
Service points won	64.7	63.4	64.2	64.8	64.0	64.1	65.8	64.3	64.3	64.4
	(0.6)	(0.6)	(0.6)	(0.6)	(0.6)	(0.6)	(0.6)	(0.6)	(0.6)	(0.2)
1st service in	58.9	60.2	58.4	58.6	59.1	59.1	59.7	60.3	61.0	59.4
	(0.6)	(0.6)	(0.6)	(0.6)	(0.6)	(0.6)	(0.6)	(0.6)	(0.6)	(0.2)
(b) Women's singles										
Aces	2.4	2.5	3.4	3.2	3.8	2.7	3.7	3.7	2.1	3.1
	(0.3)	(0.3)	(0.3)	(0.3)	(0.3)	(0.3)	(0.3)	(0.3)	(0.3)	(0.1)
Double faults	6.7	5.4	5.8	5.2	5.6	4.0	5.2	5.4	6.4	5.5
	(0.5)	(0.5)	(0.4)	(0.4)	(0.4)	(0.3)	(0.4)	(0.4)	(0.4)	(0.1)
Service points won	56.2	56.3	55.9	54.8	58.4	56.2	57.7	55.9	53.3	56.1
•	(0.9)	(1.0)	(0.8)	(0.8)	(0.8)	(0.8)	(0.9)	(0.9)	(0.9)	(0.3)
1st service in	58.3	61.0	61.3	56.9	61.1	61.9	61.9	63.9	61.4	60.8
	(0.9)	(1.0)	(0.8)	(0.8)	(0.8)	(0.8)	(0.8)	(0.9)	(0.9)	(0.3)

During the five minutes of warming up before the match begins, the same balls are used as in the first seven games. Thus it makes sense to set the age of the balls in the first game of the match at 3. If the hypothesis that new balls provide an advantage were true, the dominance of service, measured by the probability of winning a point on service, would decrease with the age of the balls. Table 13.5 does not support this hypothesis, at least in the men's singles. For the women the probability of winning a point on service with balls of age 9 is significantly lower than with balls of age 1, but overall there is no evidence for the hypothesis either.

Although serving with new balls appears to provide no advantage in terms of the number of points won, Table 13.5 shows that new balls may affect the *way* that points are won. For example, the probability of "1st service in" seems to increase when the balls become older, and the probability of a double fault seems to decrease, which is, of course, partly due to the increasing trend in the probability of "1st service in." The reason for this is that older balls are softer and fluffier (hence have more grip) than newer balls. The service is therefore easier to control, resulting in a higher percentage of "1st service in" and fewer double faults.

Both effects would result in a greater dominance of service as balls become older. To show why, nevertheless, the dominance of service appears to be independent of

the age of the balls, we split the probability of winning a point on service as follows:

D.	(maint		~ **	comica)
PI	point	won	on	service)

**TABLE 13.6** 

= Pr (point won on 1st service)	
+ Pr (1st service fault) Pr (point won on 2nd service)	
= Pr (point won if 1st service in) Pr (1st service in)	
+ $\{1 - Pr(1st \text{ service in})\}$ Pr(point won if 2nd service in)	
$\times$ Pr (2nd service in)	(13.3)

To analyze how these probabilities depend on the age of the balls, we specify a simple logit model with a linear function of the age of the balls as the systematic part; see McFadden (1984). For example, the probability of winning a point on service is specified as

$$Pr(point won on service) = \Lambda(\beta_0 + \beta_1 \times age of balls), \quad (13.4)$$

where  $\Lambda$  is the logistic distribution function,  $\Lambda(x) := \exp(x)/\{1 + \exp(x)\}$ . Table 13.6

Service characteristics depending on the age of the balls ("age"): logit estimation results.

 Probability
 Men's singles
 Women's single

Probability	Men's singles		Women'	's singles
	constant	age	constant	age
Point won on service	0.580	0.003	0.269	-0.005
	(0.019)	(0.003)	(0.027)	(0.005)
Point won on 1st service	-0.293	0.007 †	-0.593	0.019†
	(0.018)	(0.003)	(0.027)	(0.005)
Point won if 1st service in	1.001	0.002	0.443	0.011
	(0.027)	(0.005)	(0.035)	(0.006)
1st service in	0.341	$0.008^{+}$	0.340	0.020†
	(0.019)	(0.003)	(0.027)	(0.005)
Point won on 2nd service	0.057	-0.001	0.041	$-0.036^{+}$
	(0.029)	(0.005)	(0.042)	(0.008)
Point won if 2nd service in	0.414	-0.006	0.378	-0.043†
	(0.031)	(0.006)	(0.046)	(0.008)
2nd service in	1.766	0.017†	1.820	-0.001
	(0.041)	(0.008)	(0.061)	(0.011)

†: Estimate significantly different from zero.

presents the maximum likelihood estimation results for all probabilities in Equation (13.3).

As already suggested by Table 13.5, the probability of "1st service in" increases when balls become older. One might argue that this positive effect on the probability of winning a point on the first service is counteracted by a benefit for the receiver when balls become older and thus softer and fluffier. The first service would be slower and hence easier to return. We find no evidence for this, as age has no effect on the probability of winning a point if the first service is in. Therefore, in total, players win more points on their first service as balls become older.

The second service is different. The men miss fewer second services when using old balls, which is in line with the decreasing double fault statistics in Table 13.5. However, if the second service is in, they win fewer points, but not significantly so. On balance, the quality of the second service, measured by the probability of winning a point on the second service, is independent of the age of the balls.

For the women the quality of the second service *does* depend on the age of the balls. The second service is easier to return with older balls, which makes the quality of the second service depend negatively on the age of the balls.

Equation (13.3) can now be used to show that, on balance, the age of the balls does not affect the dominance of service. It is true that older balls lead to more points won on the first service. However, both men and women have fewer opportunities to score points on their second service, as the probability of missing a first service decreases. Moreover, the women score fewer points on their second service. On balance, the effects on the first and second service offset each other, so the age of the balls does not affect the quality of service.

A second interpretation of the question whether serving with new balls provides an advantage is that newer balls may benefit the server only in the first game that they are used (age = 1). It may be the transition from old, soft and fluffy balls to new, hard balls that is difficult to cope with for the receiver and/or server. To analyze this we add a dummy variable of balls of age 1 to the logit models used above. However, there is no evidence for an effect of this dummy on the probabilities in Table 13.6. Only for the probability of "2nd service in" for the womens singles does the dummy have a significantly negative effect. This is in line with the high percentage of double faults with new balls in Table 13.5. Including the dummy does not change the effect of the age variable essentially.

In summary, we have looked at three commonly held beliefs, namely that a player is as good as his/her second service, and that serving first in a set or serving with new balls provides an advantage. All three are myths and have no statistical foundation.

#### 13.4 WINNING MOOD

A big issue in the statistical analysis of sports is whether there exists a "winning mood?" This relates to the question whether subsequent points are dependent (not whether they are identically distributed; this is taken up in Section 13.5.) In this section we consider four commonly heard statements that relate to the possible existence of a winning mood in tennis, first two statements concerning the final set and then two statements regarding breaks.

## 13.4.1 At the beginning of a final set, both players have the same chance of winning the match

The "final" set—the fifth set in the men's singles (at least in a "Grand Slam" event) and the third set in the women's singles—decides a tennis match. Such a deciding set occurs in about one-fourth of all matches at Wimbledon. Tension is high and mistakes can be costly. There are a number of interesting questions related to the final set. For example, suppose that a "seed" plays against a "non-seed." You wish to forecast the winner. At the beginning of the match, if no further information is available, the relative frequency at Wimbledon that the non-seed wins is small. What is the probability at the beginning of the final set? Is it now close to 50%? Also, is it more difficult for unseeded women to beat a seed than for men, and are men more equal in quality than women? Do players get tired, i.e. does the dominance of the service decrease in long matches (i.e. in the final set). Finally, is it true that, in the final set, the player who has won the previous set has the advantage? Some of these questions will be investigated in this section.

Suppose that a seed plays against a non-seed. At the beginning of the match, if no further information is available, the probability that the non-seed will win is small: 13.1% (2.7%) in the men's singles and 10.5% (2.5%) in the women's singles. (The figures in parentheses are the standard errors.) What is the probability at the beginning of the final set? Some people claim that at the beginning of a final set, both players have the same chance of winning the match.

This is not true. Naturally, the probabilities have increased, but only to 28.7% (8.7%) for the men and 17.1% (6.3%) for the women. Both are significantly different from 50%. The reader may argue that these estimates are biased upwards because of a selection effect (a player ranked 17 is more likely to play and win in a final set against one of the top-16 players, than a player ranked 50), and that this bias is absent in the estimates 13.1% and 10.5% above. The reader would be right, but our conclusion is based on the fact that 28.7% and 17.1% are both significantly smaller than 50%, and this will be even more true if both percentages become smaller after properly accounting for the selection effect. At 2–2 in sets in the men's singles, therefore, it is certainly not true that the chances are now even between the seed and the non-seed. The seed is still very much the favorite. This is even clearer in the women's singles. At 1–1 in sets, the seeded player still has a probability of 82.9% (6.3%) of winning the match!

At first sight, the estimated probabilities of winning the final set—28.7% for the men and 17.1% for the women—also seem to indicate that it is more difficult for a female non-seed to beat a seed than is the case for a man, as is often claimed. However, one should keep in mind that the male non-seed has to win two sets to arrive at the final set, whereas the woman has to win only one set. (The fact that the tennis scoring system has an impact on the probability of winning a match was investigated in Jackson, 1989; Maisel, 1966; Miles, 1984; Riddle, 1988; and Riddle, 1989.) Therefore, when arriving at the final set, the quality difference between the non-seed and the seed is generally smaller for the men than for the women. This in itself will result in a higher probability of winning the final set for unseeded men

than for unseeded women.

## 13.4.2 IN THE FINAL SET THE PLAYER WHO HAS WON THE PREVIOUS SET HAS THE ADVANTAGE

We next consider the relation between the final and "pre-final" sets. One often hears that in the final set, the player who has won the previous set has the advantage. Is this true? We investigate this hypothesis by looking at probabilities of winning

#### **TABLE 13.7**

Estimated probabilities of winning fifth (third) set after winning fourth (second) set in men's (women's) singles.

	Probability (%)						
	Sd-Sd	Sd-NSd	NSd-Sd	NSd-NSd	Total		
Men's singles	10.7	40.8	76.4	57.6	50.2		
	(10.3)	(11.3)	(15.0)	(12.8)	(7.0)		
Women's singles	33.7	62.3	14.8	68.5	61.2		
-	(15.8)	(8.9)	(14.5)	(13.4)	(6.5)		

the final set after winning the "pre-final" set. These probabilities are presented in Table 13.7 for all matches taken together (Total) and for different types of match. Sd-NSd indicates again a match between a seeded player and an unseeded player, where the first player (Sd) won the "pre-final" set. Sd-Sd, NSd-Sd and NSd-NSd are similarly defined. Because the number of observations is quite small (51 final sets in the men's singles and 57 final sets in the women's singles in our data set), the standard errors are quite large.

In the men's singles, the probability that the same player wins the fourth and fifth sets is estimated to be 50.2% (7.0%). In the women's singles, the estimated probability that the same player wins the second and third sets is 61.2% (6.5%). These percentages are not significantly different from 50%, so there appears to be no basis for the hypothesis in question.

If we look at the subcategories, then we see that, when two seeds play against each other, the winner of the fourth (second) set will probably lose the match, especially in the men's singles. When an unseeded woman plays against a seed, winning the "pre-final" set is also a disadvantage, because her probability of winning the match is only 14.8% (14.5%). Both results are significant and indicate that, if there is correlation between the final and "pre-final" sets, then this is more likely to be negative than positive.

#### AFTER BREAKING YOUR OPPONENT'S SERVICE THERE IS AN INCREASED 13.4.3 CHANCE THAT YOU WILL LOSE YOUR OWN SERVICE.

The server is expected to win his or her service game, particularly on fast surfaces such as grass. If he or she fails to do so, this is considered serious in women's singles and disastrous in men's singles. A game not won by the server but by the receiver is called a "break."

As we show in Klaassen and Magnus (2001), points in a tennis match are not independent. Maybe the same holds at game level. If so, this dependence of games would have important consequences for the statistical modeling of tennis matches. We now ask whether games are independent, and, if so, where this comes from. One source of dependence of games could be the often heard "break-rebreak" effect: After breaking your opponent's service there is an increased chance that you will lose your own service.

At first sight there appears to be no support for the "break-rebreak" effect in Table 13.8. Overall, the probability of winning a service game increases rather than decreases after a break, in the men's singles from 80.1% to 83.4%, in the women's singles from 61.2% to 66.9%. The increase is 3.3%-points (1.1%) for the men and 5.7%points (1.6%) for the women, hence both are significant. The reader may protest by

happened in the previous game (same set).							
	Previous game	Sd-Sd	Sd-NSd	NSd-Sd	NSd-NSd	Total	
Men	Break	86.7	87.0	73.5	83.5	83.4	
		(3.0)	(1.4)	(2.6)	(1.8)	(1.0)	
	No break	85.6	89.4	74.3	78.6	80.1	
		(1.2)	(0.7)	(0.9)	(0.9)	(0.5)	
Women	Break	67.5	77.7	50.6	65.6	66.9	
		(3.5)	(1.7)	(2.8)	(2.9)	(1.3)	
	No break	65.4	77.8	48.3	61.2	61.2	
		(2.5)	(1.6)	(1.6)	(2.2)	(1.0)	

#### **TABLE 13.8**

Estimated probabilities of winning a service game depending on what

arguing that we should only consider matches between players of equal strength, Sd-Sd or NSd-NSd, because with players of unequal strength (Sd-NSd and NSd-Sd) it is to be expected that the stronger player often breaks the weaker player and then goes on to win his/her own service game, whether a "break-rebreak" effect exists or not. However, even if we constrain ourselves to matches between players of equal strength, then the "break-rebreak" effect is still absent, as Table 13.8 shows. Hence, our conclusion is unchanged: after a break, the players are more, rather than less, likely to win their service game. Thus, in the game following a break, it is not true that the winner takes it a bit easier and the loser tries a bit harder. Apparently,

what happens is just the opposite. The winner gains in confidence and the loser gets discouraged. So there is positive correlation across games. Note that this effect is stronger in NSd-NSd matches than in Sd-Sd matches. Seeds appear to play less dependently than non-seeds.

## 13.4.4 AFTER MISSING BREAK POINTS IN THE PREVIOUS GAME THERE IS AN INCREASED CHANCE THAT YOU WILL LOSE YOUR OWN SERVICE

Suppose now that in the previous game no break has occurred, but that the receiver had a good chance to break because he/she had one or more break points. The receiver did not, however, capitalize on these break points and this may have discouraged him/her. Does such an effect occur, and does it affect the current game? That is, is it true that after missing one or more break points in the previous game there is an increased chance that you will lose your own service? Table 13.9 shows no support

#### **TABLE 13.9**

Estimated probabilities of winning a service game depending on what happened in the previous game (same set).

TT	8					
	Previous game	Sd-Sd	Sd-NSd	NSd-Sd	NSd-NSd	Total
Men	Break point(s), but no break No break points	87.7 (3.1) 85.3 (1.3)	91.8 (1.6) 89.0 (0.8)	70.5 (2.8) 74.8 (1.0)	76.3 (2.6) 78.9 (1.0)	79.0 (1.3) 80.2 (0.5)
Women	Break point(s), but no break No break points	61.3 (5.6) 66.4 (2.7)	75.2 (3.4) 78.5 (1.8)	48.7 (4.3) 48.3 (1.7)	50.9 (5.6) 62.8 (2.4)	56.9 (2.4) 61.9 (1.1)

for this idea for the men, as the probability of a break is only slightly higher than usual: 1.2%-points (1.4%). For the women, however, the increase is 5.0%-points (2.6%), so missing break points seems to affect the next game. Women play the games more dependently than men. More specifically, there is a positive correlation across games: discouragement because of missed break points causes problems in the next game.

The "discouragement effect" is particularly strong when two non-seeds play against each other. There the probability of losing a service game after missed break points in the previous game increases by 11.9%-points (6.1%) in the women's singles. This adds support to the idea that seeded players are not only technically but also mentally stronger than non-seeded players. They don't let themselves be discouraged and are less affected by what happened in the previous game. This is also supported by the men's percentages.

Hence, games in a tennis match appear not to be independent. More specifically,

there is a positive correlation across games: if you did well in the previous game, you will do well in the current game. For example, after breaking your opponent's service, you are in a "winning mood" and are more likely to win your own service game. Moreover, if you had break points in the previous game but missed them all, it is more difficult to win your own service game, at least for the women, not for the men. The correlation across games is more present in matches between two non-seeds than in matches between two seeds, thus supporting the idea that seeded players are not only technically but also mentally stronger than non-seeds. These results at game level correspond to the positive correlation at point level found in Klaassen and Magnus (2001).

#### 13.5 **BIG POINTS**

Few readers would disagree that a point played at 30–40 in a tennis game is more important than, say, at 0–0. Similarly, a game at 5–4 in a set is more important than a game at 1–0. And the final set is more important than the first set. The question which concerns us here is whether players play each point "as it comes" or whether they are affected by what happened in the previous point (ace, double fault) and/or by the importance of the point (break point, game point). Do players make sure their first service is in at break point or game point? Do the "real champions" play their best tennis at the big (important) points, or do big points not exist? Do "big games" exist, for example the seventh game? If there are big points and if players do not play each point as it comes, then clearly points are not identically distributed and this will have important consequences for the statistical modeling of tennis matches (see also Klaassen and Magnus (2001)).

#### 13.5.1 The seventh game

Our first myth is not about big points but about big games, and it is a commentators' favorite—the idea that the seventh game is especially important. We don't know the history of this "wisdom," but there is no doubt that many believe it. We can measure the importance of a game in two ways, unweighted or weighted. The unweighted importance of a game in a set is defined as the probability that the server in that game wins the set given that he/she wins the game minus the probability that the server wins the set given that he/she loses the game. (Note that if we had defined the unweighted importance of a game in a set as the probability that the server in that game wins the set given that he/she wins the game (so not minus the other probability), then we would not have taken account of the fact that, after the first set, in odd games usually the player who won the previous set (often the weaker player) serves. This would have biased the importance measure.) The unweighted importance does not take into account that some games (like the 12th) occur much less frequently than other games (such as the first 6, which occur in every set). The weighted importance takes this into account, since the weighted importance is defined as the unweighted importance times the probability that the game occurs.

As Table 13.10 shows, the idea that the 7th game is singled out to be important

TABLE 13.10		
(Un)Weighted importance	e of each game in a	a set.

	Men's singles					Women's singles			
Game	Unweighted		Weighted		Unweighted		Weighted		
		-		-		•			
1	36.4	(3.5)	36.4	(3.5)	27.1	(4.5)	27.1	(4.5)	
2	31.9	(3.7)	31.9	(3.7)	42.5	(4.2)	42.5	(4.2)	
3	40.4	(3.2)	40.4	(3.2)	39.0	(4.1)	39.0	(4.1)	
4	40.8	(3.5)	40.8	(3.5)	40.0	(4.2)	40.0	(4.2)	
5	35.9	(3.5)	35.9	(3.5)	38.1	(4.3)	38.1	(4.3)	
6	44.0	(3.4)	44.0	(3.4)	33.1	(4.3)	33.1	(4.3)	
7	39.4	(3.3)	38.6	(3.3)	34.9	(4.4)	32.7	(4.2)	
8	30.1	(4.3)	26.6	(4.0)	47.4	(4.7)	36.8	(4.0)	
9	36.9	(4.0)	27.8	(3.3)	34.2	(5.6)	19.5	(3.7)	
10	58.2	(4.1)	31.6	(2.6)	55.4	(6.7)	20.2	(3.5)	
11	57.0	(4.6)	15.7	(1.6)	39.8	(10.6)	6.0	(2.4)	
12	67.7	(4.4)	18.9	(1.6)	67.1	(7.9)	10.0	(1.9)	
Mean	43.2	(1.1)	32.4	(0.9)	41.6	(1.7)	28.8	(1.1)	

is nonsense. Unweighted, games 10, 11, and 12 in the men's singles and games 10 and 12 in the women's singles are the most important. There is no indication that the 7th game plays a special role. Since the 10th, 11th and 12th games don't occur as frequently as the first 6 games, their weighted importance is rather less. Weighted, the 7th game does not play a special role either. Hence we reject this hypothesis. In addition, Table 13.10 confirms again that losing a service game in the men's singles in more serious than in the women's singles, because the average importance of a game in the men's singles is 32.4 compared to 28.8 in the women's singles.

#### 13.5.2 DO BIG POINTS EXIST?

Let us now look more directly at the existence of big points. Is it true that all points are equally important, or do big points exist?

We can measure the strength of a seeded player relative to a non-seeded player by  $p_{Sd} - p_{NSd}$ , where  $p_{Sd}$  denotes the estimated probability that the seed wins a point while serving against a non-seed and  $p_{NSd}$  denotes the estimated probability that the non-seed wins a point while serving against a seed. For the men,  $p_{Sd} = 69.3\%$  and  $p_{NSd} = 61.1\%$ , whereas for the women  $p_{Sd} = 62.9\%$  and  $p_{NSd} = 50.1\%$ ; see Table 13.3. Hence,

 $p_{Sd} - p_{NSd} = \begin{cases} 8.2\% & \text{for the men} \\ 12.8\% & \text{for the women.} \end{cases}$ 

The difference between seeds and non-seeds is thus greater in the women's singles than in the men's singles, which is no surprise. There is a second way to measure the difference in strength, namely the estimated probability that the seed wins a set from the non-seed. This is 75.1% in the men's singles and 82.0% in the women's singles

and we reach the same conclusion. In Table 13.11 we present the two measures for

non-seeded player.									
Set	Men	's singles	Women's singles						
	$p_{Sd} - p_{NSd}$	Pr(Sd wins set)	$p_{Sd} - p_{NSd}$	Pr(Sd wins set)					
1	9.6	79.9	11.7	78.1					
	(1.0)	(3.2)	(1.0)	(3.4)					
2	8.8	78.2	14.7	85.6					
	(1.0)	(3.3)	(1.1)	(2.9)					
3	7.6	70.1	10.7	82.9					
	(1.0)	(3.6)	(2.1)	(6.3)					
4	7.3	70.7							
	(1.4)	(5.3)							
5	4.4	71.3		_					
	(2.1)	(8.7)							
Total	8.2	75.1	12.8	82.0					
	(0.5)	(1.8)	(0.7)	(2.1)					

**TABLE 13.11**Two measurements of the strength of a seeded player relative to a<br/>non-seeded player.

*Note:*  $p_{Sd}$ : estimated probability of seeded player winning a point on service versus a non-seeded player;  $p_{NSd}$ : estimated probability of non-seeded player winning a point on service versus a seeded player.

each set. We see that the difference in strength between a seeded and a non-seeded player (as measured by  $p_{Sd} - p_{NSd}$  decreases gradually in the men's singles, but not in the women's singles. However, in both the men's singles and the women's singles, the difference in strength is least in the final set (4.4% in the men's singles, 10.7% in the women's singles). This is what one would expect. In the deciding final set we don't expect a lot of difference in strength any more, even though a seed and a non-seed play each other.

Now a curious phenomenon occurs. Even though the relative number of points won by the seed is lowest in the final set, this is not true for the probability that the seed wins the set. The seed still has a large chance to win the final set. And this is true both for the men and for the women; see Section 13.4.1. There is only one possible explanation, and that is that some points are more important than others and that the seeds play the important points better (or equivalently, that the non-seeds play them worse). Thus, big points do exist.

#### 13.5.3 REAL CHAMPIONS

We have just seen that seeds play the big points in the final set better than non-seeds. Is this true for the whole match? Many people believe this, as we often hear that real champions play their best tennis at the big points.

Let us think of all break points as big points, a drastic simplification. When a seed serves in the men's singles, the probability of winning a point on service is 68.3% (1.2%) at break point down and 68.7% (0.3%) at other points (seeded and non-seeded receiver taken together). The difference is not significant. However, when a non-seed serves, the difference is significant: 59.2% (0.9%) at break point and 63% (0.3%) at other points. Therefore, real champions perform better at break point down than other players. However, real champions do not win more points at break point down. It is the non-seeds that win less points at break point down. So, it seems that the seeded players do not play their best tennis, but rather that the non-seeded players play their worst. Another interpretation is that the seeded server, break point down, performs better and so does the receiver (seeded or non-seeded), and that these two improvements cancel each other. In contrast, a non-seeded server, break point down, does not perform better, while the receiver (seeded or non-seeded) does.

In contrast to the men, we find no evidence for our hypothesis for the women. In the women's singles, the probability of winning a point on service is 58.2% (1.4%) at break point down and 61.6% (0.4%) at other points for a seed (seeded and non-seeded receiver taken together). For a non-seed these are 50.8% (1.1%) at break point down and 54.9% (0.4%) otherwise. Both differences are significantly negative, and the decrease for the seeds (3.4%-points (0.6%)) is only marginally smaller than that of the non-seeds (4.1%-points (1.2%)). So real champions do not perform better than others in the women's singles.

The word "real champion" can also be interpreted in a relative, rather than an absolute, manner. In an absolute sense, a player is a real champion if he/she is very good, for example a seed, irrespective of the opponent. In a relative sense, a player is a real champion if he/she performs particularly well at important points depending on the opponent.

To analyze whether (relative) real champions play their best tennis at break points, we use Table 13.12. The differences in this table show the relative service quality at break point down with respect to other points. If the hypothesis were true, a server would perform better against a non-seed than against a seed using the differences as quality measures. For the men we see that a seed performs relatively better against a non-seed than against a seed, but the difference of 1.7 percentage points (0.1 - 1.6) is insignificant. However, a non-seed plays relatively worse against a non-seed than against a seed (again insignificantly). So we find no evidence for our hypothesis in relative sense for the men. The same holds for the women.

Let us now compare the results for absolute and relative champions with the conclusion at the end of the discussion of the hypothesis about the existence of big points. Table 13.11 concerns only matches between a seed and a non-seed. So, a distinction between absolute and relative real champions is no longer possible, since seeds are champions both in absolute and relative sense compared to non-seeds.

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2stimated probabilities of winning a point of service.							
		Sd-Sd	Sd-NSd	NSd-Sd	NSd-NSd	Total	
Men	At break point	65.5	69.4	57.8	59.8	60.9	
	-	(2.2)	(1.4)	(1.2)	(1.3)	(0.7)	
	At other points	67.1	69.3	61.4	64.1	64.7	
	-	(0.6)	(0.4)	(0.4)	(0.4)	(0.2)	
	Difference	-1.6	0.1	-3.6	-4.3	-3.8	
		(2.3)	(1.5)	(1.3)	(1.4)	(0.7)	
Women	At break point	58.2	58.2	48.8	51.6	52.3	
		(2.2)	(1.7)	(1.3)	(1.9)	(0.9)	
	At other points	56.7	63.4	50.3	56.4	56.6	
		(0.8)	(0.5)	(0.5)	(0.7)	(0.3)	
	Difference	1.5	-5.2	-1.5	-4.8	-4.3	
		(2.3)	(1.8)	(1.4)	(2.0)	(0.9)	

Estimated	probabilities	s of winning	a noint	on service
Estimated	probabilities	s or winning	, a point	on service.

Therefore, we aggregate the results regarding our hypothesis across absolute and relative champions. Then our conclusion is that male seeds play break points better than non-seeds. This is in line with the conclusion from Table 13.11 that seeds play the important points in the final set better than non-seeds. Therefore, our overall conclusion for the men is that our hypothesis is true. For the women we did not find evidence for our hypothesis in terms of absolute and relative champions. However, Table 13.11 shows that seeds play the key points in the third set better than non-seeds. So, our overall conclusion for the women is that our hypothesis might be true.

#### 13.6 CONCLUSION

The television commentator at a tennis match is not to be envied. Repeating the names of the players (as football commentators do) is hardly useful. The score is typically presented in the screen and mentioning it is thus superfluous. So, what must they talk about? A match at Wimbledon in the men's singles lasts, say, two-and-a-half hours. The effective time-in-play for one point is about five seconds. With six points per game, a game lasts thirty seconds. With ten games per set, a set lasts five minutes. With four sets per match, a match lasts only twenty minutes. The remaining two hours and ten minutes are to be filled by the commentators. It follows that they play a major part in the broadcast. But are the things that they say correct? Are the idées reçues of the commentators true? Unfortunately, as we have found out in this chapter, almost none of them are.

#### **ACKNOWLEDGEMENTS**

We are grateful to the Royal Statistical Society and to Taylor and Francis Ltd. for permission to use material published earlier by us in *The Statistician* (volume 48, Part

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**TABLE 13.12** 

2, pp. 239–246 and 247–256, 1999) and the *Journal of Applied Statistics* (volume 26(4), pp. 461–468, 1999). As always we thank IBM UK and The All England Club at Wimbledon for their kindness in providing the data.

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